



The University of Fiji

SCHOOL OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF MATHEMATICS

FINAL EXAMINATION SEMESTER II, 2015

FOUNDATION MATHEMATICS II

MTHF12

Time Allowed: Three hours (3 hours)

(An extra ten minutes is allowed for reading this paper)

INSTRUCTIONS

- 1.0 Write **all** your answers in the **Answer Book** provided.
- 2.0 There are four (4) pages of questions in this Question Booklet (excluding the Cover Page)
- 3.0 Write your **Student Identification Number** on the front page of the **Answer Book**.
- 4.0 If you use extra sheets of paper, be sure to show clearly the question number(s) being answered and to tie each sheet securely in your Answer Book at the appropriate place.
- 5.0 There are **ten** questions in this paper and all are **compulsory**.
- 6.0 **Programmable calculators** are not allowed.
- 7.0 Start a new question on a new page, numbering them clearly on the margin.

QUESTION 1 [Start the question on a new page]

- (a) Evaluate the summation notation.

$$\sum_{r=16}^{50} (3-2r)^2$$

- (b) A bob of a pendulum swings through an arc of 50 cm on its first swing. Each successive swing is 90% of the length of the previous swing. Find the total distance the bob travels before coming to rest.
- (c) A display of cans on a grocery shelf consists of 20 cans on the bottom, 18 cans in the next row, and so on in an arithmetic sequence, until the top row has 4 cans. How many cans, in total, are in the display?

[3 + 3 + 4 = 10 marks]

QUESTION 2 [Start the question on a new page]

- (a) A sequence is defined by $a_n = \frac{2n}{n+1}$ for any natural number, n .
- (i) Write down the first **four** terms of the sequence of partial sums associated with a_n .
- (ii) Determine whether the sequence is increasing or decreasing by comparing a_{n+1} and a_n .
- (iii) Prove that the sequence is **bounded above** by 2.
- (b) Sketch the graph of the trigonometric function on the indicated domain.
 $y = 3 \sin 4x + 2$ for $0 \leq x \leq 2\pi$.

[1 + 3 + 2 + 4 = 10 marks]

QUESTION 3 [Start the question on a new page]

- (a) Prove the trigonometric identity
$$\frac{\sin x}{1 + \cos x} + \frac{1 + \cos x}{\sin x} = 2 \operatorname{cosec} x$$
- (b) Express $2 \cos \theta - 3 \sin \theta$ in the form $R \cos(\theta - \alpha)$.
- (c) Solve the trigonometric equation where $\theta \in [0, 2\pi]$
 $2 \sin^2 \theta - \sin \theta - 1 = 0$.

[4 + 3 + 3 = 10 marks]

QUESTION 4 [Start the question on a new page]

(a) Show that the exact value of $\tan 22.5^\circ$ is $\sqrt{2} - 1$.

(b) Use partial fraction to evaluate the integral.

$$\int \frac{5x+12}{x^2+3x} dx$$

(c) Calculate the **area** bounded by the $f(x) = 4 - x^2$ and the straight line $y = -x + 2$.

[3 + 3 + 4 = 10 marks]

QUESTION 5 [Start the question on a new page]

(a) Evaluate the integral $\int \frac{x}{\sqrt{1-4x^2}} dx$ by using the **substitution** $u = 1 - 4x^2$.

(b) Use **Integration by Parts** to evaluate the integral:

$$\int x^2 \sin x dx$$

(c) Find the **mean value** of the function $y = 2 \sin 3x$ over the interval $\left[0, \frac{\pi}{3}\right]$.

[3 + 4 + 3 = 10 marks]

QUESTION 6 [Start the question on a new page]

(a) (i) Derive the formula $\frac{d}{dx} [\sec^{-1} x] = \frac{-1}{x\sqrt{x^2-1}}$.

(i) Use the formula given in part (i) above to differentiate $y = \sec^{-1} x^3$.

(b) Evaluate the integral $\int \frac{\sqrt{9-x^2}}{x^2} dx$ using the trigonometric substitution $x = 3 \sin \theta$.

(Give your answer in terms of x .)

[3 + 2 + 5 = 10 marks]

QUESTION 7 [Start the question on a new page]

- (a) Solve the differential equation for its particular solution.

$$\frac{dy}{dx} = x^3 - 2x, \quad y = 1 \text{ when } x = 0.$$

- (b) Newton's empirical law of cooling of an object is given by the differential equation

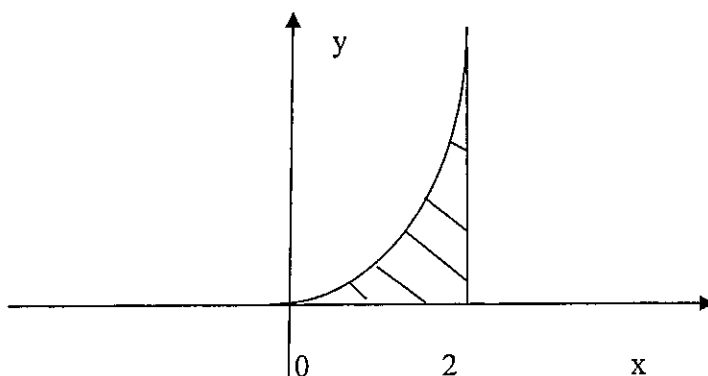
$$\frac{dT}{dt} = \alpha(T - T_m) \text{ where } T_m \text{ is the room temperature. When a chicken is removed from an}$$

oven, its temperature is measured at 300°F . Three minutes later its temperature is 200°F .

How long will it take for the chicken to cool off to 100°F given the room temperature of 70°F ? [5 + 5 = 10 marks]

QUESTION 8 [Start the question on a new page]

- (a) The region bounded by the curve $y = x^3$ on the interval $0 < x < 2$ is rotated 360° about the y -axis.



Find the **volume of the solid generated** (in cubic units) when the shaded region is rotated about the y -axis through 360° .

- (b) The mean for a sample of 16 infants was found to be 5.98 mg/dl bilirubin level of 4-day-old infants. Assuming bilirubin levels in 4-day-old infants are approximately normally distributed with a standard deviation of 3.5 mg/dl. Construct a 95% confidence interval for population mean, μ .

- (c) Determine the sample size required from Internet Usage of households in one week with a standard deviation of 6.95 minutes to allow its mean to be estimated within 1 minute with 95% confidence.

[5 + 2 + 3 = 10 marks]

QUESTION 9 [Start the question on a new page]

- (a) 50 smokers were questioned about the number of hours they sleep each day. The smokers need less sleep than the general public which needs an average of 7.7 hours of sleep with a standard deviation of 0.5 hours.
- (i) Construct a test at 5% level of significance to determine whether the null hypothesis $H_0 : \mu = 7.7$ hours can be accepted given the alternative hypothesis $H_A : \mu < 7.7$. Compute a rejection region for a significance level of 5%.
- (ii) If the sample mean is 7.5 hours, what can you conclude?
- (a) If Q is the point (1, 1, 0) and R is the point (1, 6, -5), find the coordinates of a point P on the line QR given that $QP:PR = 3:2$.

- (b) Given vectors $\underline{c} = \begin{pmatrix} t \\ -2 \\ 3 \end{pmatrix}$ and $\underline{d} = \begin{pmatrix} 2 \\ 10 \\ t \end{pmatrix}$, find the value of t if $\underline{c}$ and $\underline{d}$ are orthogonal.

[2 + 3 + 3 + 2 = 10 marks]

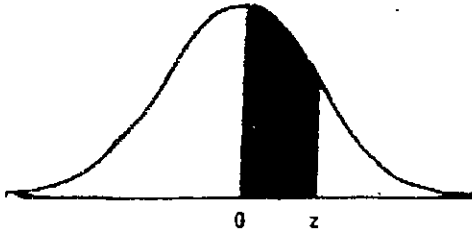
QUESTION 10 [Start the question on a new page]

- (a) Vectors $\underline{p}$, $\underline{q}$ and $\underline{r}$ are defined by $\underline{p} = i + j - k$, $\underline{q} = i + 4k$ and $\underline{r} = 4i - 3j$.
- (i) Express $\underline{p} - 2\underline{q} + 2\underline{r}$.
- (ii) Find $\underline{p} \cdot \underline{r}$.
- (iii) Find $|\underline{r}|$.
- (b) Determine the angle between the vectors $\underline{a} = \langle 3, -4, -1 \rangle$ and $\underline{b} = \langle 0, 5, 2 \rangle$.
- (c) Find symmetric equations for the line passing through the point (0, 1, 2) and parallel to the vector $\underline{a} = 6i + 3j + 2k$.

[2 + 1 + 1 + 3 + 3 = 10 marks]

THE END

AREAS UNDER NORMAL PROBABILITY CURVE



The tabulated value is the probability that the standardized normal variate Z (with $\mu=0$, $\sigma=1$) lies between 0 and z .
e.g. $P(0 < Z < 1.43) = 42.36\%$

z	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359	4	8	12	16	20	24	28	32	36
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0754	4	8	12	16	20	24	28	32	36
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141	4	8	12	15	19	22	27	31	35
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517	4	8	11	15	19	22	26	30	34
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879	4	7	11	14	18	22	25	29	32
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224	3	7	10	14	17	21	24	27	31
0.6	.2258	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2518	.2549	3	6	10	13	16	19	23	26	29
0.7	.2580	.2612	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852	3	6	9	12	15	18	21	24	27
0.8	.2881	.2910	.2939	.2967	.2996	.3023	.3051	.3078	.3106	.3133	3	6	8	11	14	17	19	22	25
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389	3	5	8	10	13	15	18	20	23
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621	2	5	7	9	12	14	16	18	21
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830	2	4	6	8	10	12	14	16	19
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015	2	4	5	7	9	11	13	15	16
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177	2	3	5	6	8	10	11	13	14
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319	1	3	4	6	7	8	10	11	13
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441	1	2	4	5	6	7	8	10	11
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545	1	2	3	4	5	6	7	8	9
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633	1	2	3	3	4	5	6	7	8
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706	1	1	2	3	4	4	5	6	6
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767	1	1	2	2	3	4	4	5	5
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817	0	1	1	2	2	3	3	4	4
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857	0	1	1	2	2	2	3	3	4
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890	0	1	1	1	2	2	2	3	3
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916	0	0	1	1	1	2	2	2	2
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936	0	0	1	1	1	1	1	2	2
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952	0	0	0	1	1	1	1	1	1
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964	0	0	0	0	1	1	1	1	1
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974	0	0	0	0	0	1	1	1	1
2.8	.4974	.4975	.4976	.4977	.4978	.4979	.4979	.4979	.4980	.4981	0	0	0	0	0	0	0	0	1
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986	0	0	0	0	0	0	0	0	1
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990	0	0	0	0	0	0	0	0	0
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993	0	0	0	0	0	0	0	0	0
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4994	.4995	.4995	0	0	0	0	0	0	0	0	0
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997	0	0	0	0	0	0	0	0	0
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998	0	0	0	0	0	0	0	0	0
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	0	0	0	0	0	0	0	0	0
3.6	.4998	.4998	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	0	0	0	0	0	0	0	0	0
3.7	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	0	0	0	0	0	0	0	0	0
3.8	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	0	0	0	0	0	0	0	0	0
3.9	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	0	0	0	0	0	0	0	0	0

ORDINATES OF THE NORMAL DISTRIBUTION

z	0	1	2	3	4	5	6	7	8	9
0.	.3989	.3970	.3910	.3814	.3683	.3521	.3332	.3123	.2897	.2661
1.	.2420	.2179	.1942	.1714	.1497	.1295	.1109	.0940	.0790	.0656
2.	.0540	.0440	.0355	.0283	.0224	.0175	.0136	.0104	.0079	.0060
3.	.0044	.0033	.0024	.0017	.0012	.0009	.0006	.0004	.0003	.0002