



The University of Fiji
School of Science and Technology
Department of Mathematics

MTHF12

Foundation Mathematics II

Semester II, 2010

Final Examination Paper

Time Allowed: 3 Hours [10 Minutes Extra Allowed For Reading The Paper]
Weighting: 60 %
Total Marks: 100

Instructions

- ✓ There are 8 questions and all are compulsory.
- ✓ Start each question on a new page, numbering them clearly on the margin.
- ✓ Show your workings clearly in each question to gain maximum marks.
- ✓ You may ask for extra sheets from the supervisors for your rough working.
- ✓ Do not forget to write your name and ID number on all the extra sheet(s) attached.
- ✓ Programmable calculators are not allowed.

Good Luck !

Question 1

- (A) Compute the following summation

$$\sum_{i=1}^{75} (3i^2 - 12i + 9)$$

- (B) Given the following geometric series, write down the formula for the n^{th} term of the sequence and thus compute the 20^{th} term.

$$< 16, 24, 36, 54, \dots >$$

- (C) A 10-metre pile is to be driven into very hard ground by a pile-driver for the construction of a bridge. The first blow drives it in 1 metre and each subsequent blow drives it in $\frac{1}{6}$ of the distance of the previous blow. What length of the pile will be left above the ground no matter how many blows are applied to the pile?

- (D) Find the value to which the following sequence converges to.

$$\delta_n = \frac{3n - 1}{6n^2 + 4}$$

$$[3 + 3 + 4 + 3 = 13 \text{ Marks}]$$

Question 2

- (A) For the following series, calculate the sum of the first 13 terms using the appropriate formula of summation.

$$< 24, -18, 13.5, \dots >$$

- (B) Convert 130.4° to radians correct to 4 significant digits.

- (C) Prove the following trigonometric identity:

$$\frac{\operatorname{cosec} \alpha}{\cot \alpha + \tan \alpha} = \cos \alpha$$

- (C) Sketch the graph of the following trigonometric function on the indicated domain.

$$\varphi = 2 - \cos \theta, \quad -2\pi < \theta < 2\pi$$

[3 + 2 + 3 + 4 = 12 Marks]

Question 3

- (A) Solve the following trigonometric equation where $\phi \in [0, 2\pi]$.

$$\sec(2\phi) = -2.321$$

- (B) Evaluate the following integral.

$$\int \frac{e^{5x} - e^{2x}}{e^{3x}} dx$$

- (C) Calculate the area enclosed by the function $y = \sqrt{x}$, the line $y = x - 2$ and the y -axis.

- (D) Use the indicated u -substitution to evaluate the following integral.

$$\int \frac{x^3}{x^2 - 1} dx, \quad u = x^2 - 1$$

[3 + 3 + 4 + 3 = 13 Marks]

Question 4

- (A) Use Integration by Parts to evaluate the following integral.

$$\int x \sin x dx$$

- (B) Find the Mean Value of the function $y = x^4 - 4x^2 - x$ on the interval $[-1, 2]$.

- (C) The hyperbola $y = \frac{6}{x}$ is rotated about the x -axis between $x = 2$ and $x = 3$. Calculate the volume of the solid of revolution generated.

[4 + 4 + 4 = 12 Marks]

Question 5

- (A) Use trigonometric substitution to evaluate the following integral.

$$\int \frac{1}{\sqrt{4-x^2}}$$

- (B) Show that $x = a \cos(\omega t) + b \sin(\omega t)$ is a solution to the following differential equation.

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

- (C) Solve for the particular solution of the following differential equation.

$$\frac{dy}{dx} - xy = x, \quad y(0) = 1$$

[4 + 4 + 4 = 12 Marks]

Question 6

- (A) The atmospheric pressure, P , varies with height, h , in such a way that $\frac{dP}{dh} = -kP$, where k is a constant. At sea level ($h = 0$), the pressure is $10^5 N/m^2$ and at a height of $2000 m$, the pressure is $8 \times 10^4 N/m^2$. Find the pressure when the height is $3000 m$.

- (B) A population of ripe coconuts is known to have a standard deviation of 150 mls in the amount of juice (coconut water) found in the coconuts. A sample of 25 ripe coconuts is found to have a mean amount of 500 mls of juice. Find the 99% confidence interval for μ .

- (C) For the previous question, how large a sample is required if you want to be 95% confident that the estimate of μ is not off by more than 5×10^{-3} .

[5 + 4 + 3 = 12 Marks]

Question 7

- (A) Hypothesis testing encounters two types of errors, Type I and Type II. Differentiate between the two.

(B) What does it mean when we say that 0.05 level of significance, α , is used by a statistician in a hypothesis testing? If $\alpha = 0.05$, what would be the probability of acceptance?

(C) The mean quantity of liquid fertilizer in a sample of 75 containers produced by a chemical plant is 798 ml, with a standard deviation of 16 ml.

Assuming that the sample standard deviation is a reliable estimate of the population standard deviation, and μ is the mean quantity of liquid fertilizer in all containers, test a null hypothesis $H_0 : \mu = 800$ against the alternative hypothesis $H_1 : \mu < 800$ at a 1 % significance level. State your conclusion.

[3 + 3 + 4 = 10 Marks]

Question 8

(A) For the following vectors $\vec{p} = 3i - j + 2k$, $\vec{q} = i - 3j - 1.5k$, find whether they are parallel, orthogonal or neither of the two.

(B) Points A and B are position vectors where

$$\vec{a} = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} \quad \text{and} \quad \vec{b} = \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix}$$

- (i) Express the vector AB in terms of the unit vectors $\vec{i}$, $\vec{j}$, $\vec{k}$.
- (ii) Determine the scalar product of $\vec{a}$ and $\vec{b}$.
- (iii) Find the angle between the vectors $\vec{a}$ and $\vec{b}$.

(C) Solve for p , q , and r such that:

$$4 \begin{pmatrix} p \\ 4 \\ -52 \end{pmatrix} - 7 \begin{pmatrix} -1 \\ q \\ 7 \end{pmatrix} = 9 \begin{pmatrix} -6 \\ 6 \\ r \end{pmatrix}$$

[3 + 3 + 3 + 4 + 3 = 16 Marks]

End of Questions

P.T.O. for Normal Distribution Table