

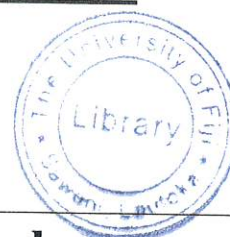


The University of Fiji
(An Entity of Arya Pratinidhi Sabha of Fiji)



SCHOOL OF SCIENCE AND TECHNOLOGY
Semester 1: Year 2025

FINAL EXAMINATION PAPER



MTH111 Calculus 1 and Linear Algebra

Important Instructions to Students:

1. Read the questions, instructions carefully and answer the questions.
2. You are allowed an **extra 10 minutes reading time** (during this time, you are not allowed to write with pen) and **180 minutes (3 hours) to complete this paper**.
3. Silent, non-programmable calculators are allowed, but not supplied.
4. Write all your answers in the answer booklet provided. If you need extra sheets, please ask from the supervisor. Write your name, course code and student identification number on the answer booklet and on the attendance sheet.
5. There are two (2) sections in this paper worth 50 marks each. Attempt all 6 questions. Start a new question on a new page.
6. Write all your answers legibly. The marker has the right to award zero for answers that are not legible.
7. This is a **Closed Book Examination**. The University views plagiarism, cheating and dishonest practice as serious offences and if found, offenders will be penalized.

Examination Paper Content (Format):

<u>Section</u>	<u>Question</u>	<u>Content</u>	<u>Marks</u>	<u>Suggested Time</u>
A	1-3	Linear Algebra	50	90 minutes
B	4-6	Calculus	50	90 minutes
		TOTAL	100	180 Minutes

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Registration Certificate Number (RGN0020/11)*

Section A: Linear Algebra**[50 marks]****Question 1** *Start this question on a new page.*

(a) Let $A = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 5 & 4 \end{bmatrix}$, and $C = \begin{bmatrix} 3 & 9 & 5 \\ 4 & 5 & 1 \end{bmatrix}$.

Compute the following:

(i) $A^T B$

(ii) $(BA)^T$

(iii) $A - 3C$

(iv) $(2A^T - C^T)$

[2+2+2+2=8 marks]

(b) Using Gaussian elimination with back substitution, solve the system of linear equations.

$$\begin{aligned} x_2 - x_3 &= 0 \\ x_1 - 3x_3 &= -1 \\ -x_1 + 3x_2 &= 1 \end{aligned}$$

[4 marks]

(c) Use Gauss Jordan elimination method to solve the system of linear equations.

$$\begin{aligned} 2x - 4y + 2z &= 2 \\ x + 2y + 3z &= 7 \\ 3x - 6y + 4z &= 4 \end{aligned}$$

[4 marks]

Question 2 *Start this question on a new page.*

(a) Consider the system of linear equations.

$$\begin{aligned} x_1 - 2x_2 - x_3 &= 1 \\ 3x_1 - 5x_2 - 2x_3 &= -2 \\ 2x_1 - 5x_2 - 2x_3 &= 0 \end{aligned}$$

(i) Write the system in the form $Ax = b$.(ii) Find A^{-1} , the inverse of A .(iii) Use A^{-1} to solve for x .

[1+5+2=8 marks]

(b) Let $A^{-1} = \begin{bmatrix} 2 & 5 \\ -7 & 6 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 7 & -3 \\ 2 & 0 \end{bmatrix}$.

Find:

(i) $(AB)^{-1}$ (ii) $(A^T)^{-1}$

(iii) A^{-2} (iv) $(2A)^{-1}$

[2+2+2+2=8 marks]

(c) Given the system of linear equations

$$2x + y = 1$$

$$y - z = 2$$

$$-2x + y + z = -2$$

(i) Find the LU – factorization of the coefficient matrix, A .

(ii) Solve the lower triangular system $Ly = b$.

(iii) Solve the upper triangular system $Ux = y$.

[3+1+1=5 marks]

Question 3 *Start this question on a new page.*

(a) Use cofactor expansion to evaluate the determinant of $A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 4 & 4 \\ 1 & 0 & 2 \end{bmatrix}$.

[2 marks]

(b) Let $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 3 & 4 \\ 0 & 0 & 1 \end{bmatrix}$.

(i) Find the characteristic equation of A .

(ii) Find the eigenvalues of A .

(iii) Find the corresponding eigenvectors of A .

[1+1+6=8 marks]

(c) Use Cramer's rule to solve the system of linear equations

$$\begin{aligned}x_1 + 2x_2 - x_3 &= 0 \\4x_1 + x_2 + 2x_3 &= 1 \\x_1 - x_2 + 3x_3 &= 4\end{aligned}$$

[3 marks]

Section B:

Calculus

[50 marks]

Question 4 *Start this question on a new page*

(a) Let $f(x) = \frac{x}{3x^2 + 1}$ and $g(x) = \sqrt{1-x}$. Find

(i) $f + g$

(ii) $f - g$

(iii) f/g

(iv) $f \circ g$

(v) $g \circ f$

(vi) State the domain of $g \circ f$

[3 marks]

(b) Consider the piecewise function

$$f(x) = \begin{cases} -x+1, & \text{if } x < 1 \\ x-1, & \text{if } 1 < x < 2 \\ 5-x^2, & \text{if } x \geq 2 \end{cases}$$

(i) Evaluate $\lim_{x \rightarrow 1} f(x)$ if it exists.

(ii) Show that f is continuous at $x = 2$.

[2+4=6 marks]

Question 5 *Start this question on a new page*

(a) Evaluate the limits.

(i) $\lim_{x \rightarrow +\infty} (\sqrt{x^6 + 5x^3} - x^3)$

(ii) $\lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 + x}}{5 + 3x}$

[5+4=9 marks]

(b) Use the techniques of differentiation to find $\frac{dy}{dx}$, the first derivative of y .

(i) $\cos x + \sin x = x^2 + y^2$

(ii) $y = \frac{2x^3}{(x^2 - 4)^2}$

[3+3=6 marks]

(c) Let $f(x) = x^3 - 3x$.

(i) What are the critical points of $f(x)$?

(ii) Find the intervals on which $f(x)$ is increasing and decreasing.

(iii) Use the first derivative test to determine the relative maximum and the relative minimum points.

(iv) Calculate the absolute maximum and the absolute minimum in the interval $[0, 2]$.

[2+4+2+2=10 marks]

Question 6 *Start this question on a new page*

(a) Let $f(x) = x^3 + x - 1$.

(i) Use the Intermediate Value Theorem to show that $f(x)$ has a root in the interval $[0, 1]$.

(ii) Starting at $x_0 = 1$, use the Newton's method to approximate this root.

[1+3=4 marks]

(b) Use logarithmic differentiation to solve for $\frac{dy}{dx}$, if $y = \frac{x^3 \sqrt[4]{8x-12}}{(1+x^2)^5}$.

[4 marks]

(c) Evaluate the integrals using appropriate substitutions.

(i) $\int x^2 e^{x^3} dx$

(ii) $\int \frac{x^2 + 1}{\sqrt{x^3 + 3x}} dx$

[4+4=8 marks]

End of Paper

