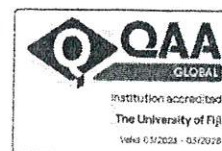




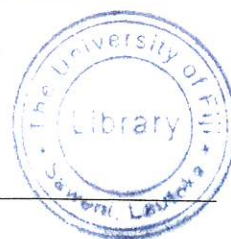
The University of Fiji
(An Entity of Arya Pratinidhi Sabha of Fiji)



SCHOOL OF SCIENCE AND TECHNOLOGY
Semester 1: Year 2025

FINAL EXAMINATION PAPER

MTH211 Advanced Calculus



Important Instructions to Students:

1. Read the questions, instructions carefully and answer the questions.
2. You are allowed an **extra 10 minutes reading time** (during this time, you are not allowed to write with pen) and **180 minutes (3 hours) to complete this paper**.
3. Silent, non-programmable calculators are allowed, but not supplied.
4. Write all your answers in the answer booklet provided. If you need extra sheets, please ask from the supervisor. Write your name, course code and student identification number on the answer booklet and on the attendance sheet.
5. Attempt all 5 questions. Each question is worth 20 marks. Start a new question on a new page.
6. Write all your answers legibly. The marker has the right to award zero for answers that are not legible.
7. This is a **Closed Book Examination**. The University views plagiarism, cheating and dishonest practice as serious offences and if found, offenders will be penalized.

Examination Paper Content (Format):

Question	Content	Marks	Suggested Time
1	Infinite Series	20	36 minutes
2	Infinite Series	20	36 minutes
3	Analytic Geometry in Calculus	20	36 minutes
4	Partial Derivatives	20	36 minutes
5	Multiple Integrals	20	36 minutes
	TOTAL	100	180 Minutes

*Registered with the Fiji Higher Education Commission as a University under the Higher Education Act 2008.
Registration Certificate Number (RGN0020/11)*

Question 1 *Start this question on a new page.*

(a) For the sequence $\left\{ \frac{n}{2n+1} \right\}_{n=1}^{+\infty}$,

(i) Use the difference $a_{n+1} - a_n$ to show that the sequence is strictly increasing.

(ii) Use the ratio a_{n+1}/a_n to show that the sequence is strictly increasing.

(iii) Use differentiation to show that the sequence is strictly increasing.

[2+2+2=6 marks]

(b) Apply the divergence test to determine whether the series converges or diverges.

$$\sum_{k=1}^{\infty} \frac{k^2 + k + 3}{2k^2 + 1}$$

[2 marks]

(c) Use the technique of telescoping sum to show that $\sum_{k=1}^{\infty} \left(\frac{2}{k(k+2)} \right) = \frac{3}{2}$.

[4 marks]

(d) Confirm that the integral test is applicable and use it to determine whether the series

$$\sum_{k=1}^{\infty} \frac{1}{5k+2} \text{ converges.}$$

[3 marks]

(e) Use the comparison test to determine whether the series $\sum_{k=1}^{\infty} \frac{k}{k^{3/2} - 1/2}$ converges or diverges.

[2 marks]

(f) Use the limit comparison test to show that the series $\sum_{k=1}^{\infty} \frac{4k^2 - 2k + 6}{8k^7 + k - 8}$ converges.

[3 marks]

Question 2 *Start this question on a new page.*

- (a) Use the ratio test to determine whether the series $\sum_{k=1}^{\infty} \frac{3^k}{k!}$ converges or diverges.

[2 marks]

- (b) Use the root test to determine whether the series $\sum_{k=1}^{\infty} \left(\frac{3k+2}{2k-1} \right)^k$ converges or diverges.

[2 marks]

- (c) Determine whether the alternating series $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k+1}{3k+1}$ converges. Justify your answer.

[3 marks]

- (d) Use the ratio test for absolute convergence to determine whether the series $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{3^k}{k^2}$ converges or diverges.

[4 marks]

- (e) Find the interval of convergence and the radius of convergence of the series $\sum_{k=0}^{\infty} \frac{x^k}{k+1}$.

[5 marks]

- (f) (i) Find the Taylor polynomial of degree 4 for the function $f(x) = \sqrt[4]{x}$ at $x = 1$.

- (ii) Apply Taylor's theorem to find an upper bound for remainder $R_4(x)$ on the interval

$$\frac{1}{2} \leq x \leq \frac{3}{2}.$$

[2+2=4 marks]

Question 3 *Start this question on a new page.*

(a) The Cartesian coordinates of a point are $(-8, 1)$. Determine a set of polar coordinates for the point.

[3 marks]

(b) Sketch the graph of $r = 4 - 9\sin\theta$ in polar coordinates.

[5 marks]

(c) Calculate the arc length of the entire cardioid $r = a(1 - \cos\theta)$.

[6 marks]

(d) Find the area of the region inside the cardioid $r = 2 + 2\cos\theta$ and outside the circle $r = 3$.

[6 marks]

Question 4 *Start this question on a new page*

(a) Determine whether the limit exists. If so, find its value.

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^4 - 16y^4}{x^2 + 4y^2}$$

[2 marks]

(b) Given $z = (2x - y)^5$, find $\frac{\partial^3 z}{\partial y \partial x \partial y}$.

[2 marks]

(c) Show that the function $z = e^x \sin y + e^y \cos x$ satisfies Laplace's equation $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$.

[4 marks]

(d) Let $T = x^2 y - xy^3 + 2$; $x = r \cos \theta$, $y = r \sin \theta$. Use the chain rule to find $\frac{\partial T}{\partial r}$ and $\frac{\partial T}{\partial \theta}$.

[5 marks]

(e) Suppose that the temperature at a point (x, y) on a metal plate is

$T(x, y) = 4x^2 - 4xy + y^2$. An ant, walking on the plate, traverses a circle of radius 5 centered at the origin. What are the highest and lowest temperatures encountered by the ant?

[7 marks]

Question 5 *Start this question on a new page*

(a) Suppose that the temperature in degrees Celsius at a point (x, y) on a flat metal plate is $T(x, y) = 10 - 8x^2 - 2y^2$, where x and y are in meters. Find the average temperature of the rectangular portion of the plate for which $0 \leq x \leq 1$ and $0 \leq y \leq 2$.

[5 marks]

(b) Evaluate the integral $\iint_R x(1+y^2)^{-1/2} dA$ over the region R where R is the region in the first quadrant enclosed by $y = x^2$, $y = 4$, and $x = 0$.

[5 marks]

(c) Find the surface area of the portion of the cylinder $y^2 + z^2 = 9$ that is above the rectangle $R = \{(x, y) : 0 \leq x \leq 2, -3 \leq y \leq 3\}$.

[5 marks]

(d) Evaluate $\iiint_G z dV$, where G is the solid in the first octant that is bounded by the cylinder $y^2 + z^2 = 9$ and the planes $x = 0$, $y = 3x$, and $z = 0$.

[5 marks]

End of Paper

